Name: SOLUTIONS Student ID:

MATH 53, QUIZ 3

Be sure to show neat, organized, complete work in the space provided.

1. This problem refers to Figure 1. Let A be the plane $2x + 2y - z = 1$ and let B be the plane $x + y + 2z = 6$.

   (a) Find a vector parallel to the line of intersection L of these two planes.

METHOD 1: take cross product of normals:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 1 & 1 & 2 \end{vmatrix} = \langle 5, -5, 0 \rangle$$

METHOD 2: Try to solve system

$$\begin{cases} z = 2x + 2y - 1 \\ x + y + 2z = 6 \end{cases}$$

$$\begin{cases} z = 2x + 2y - 1 \\ 5x + 5y = 8 \end{cases}$$

2A: Find 2 solutions, e.g. $(0, \frac{8}{5}, \frac{11}{5})$ & $(\frac{8}{5}, 0, \frac{11}{5})$

take difference: $\langle \frac{8}{5}, -\frac{8}{5}, 0 \rangle$

2B: Parametrize all solutions:

$\vec{r}(t) = \langle t, \frac{8}{5} - t, \frac{11}{5} \rangle$

which has direction vector $\langle 1, -1, 0 \rangle$.

Answer: $\langle 1, -1, 0 \rangle$ (or any nonzero multiple of this).

   (b) Find the equation of the plane C which is perpendicular to L and passes through the point P with coordinates $(3, -4, 2)$.

Have normal vec & pt:

$$(1)(x-3) + (-1)(y-(-4)) + (0)(z-2) = 0$$

$$x - 3 - y - 4 = 0$$

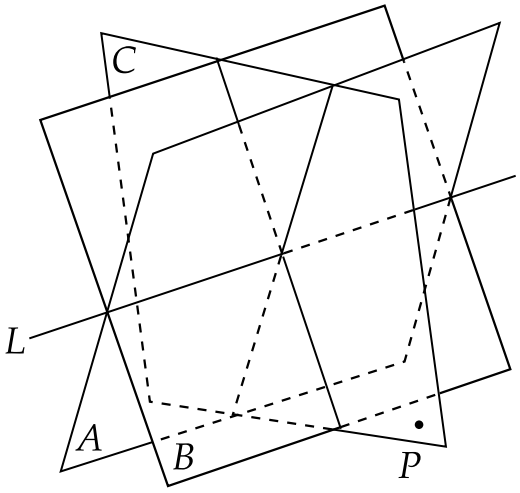

Figure 1: The planes A and B intersect in the line L. The plane C is perpendicular to L and passes through the point P.

Answer: $x - y - 7 = 0$

(Continued on back.)

2. Identify the following shapes in $\mathbb{R}^3$. Here is the list of possible answers:

   ellipsoid, elliptic paraboloid, hyperbolic paraboloid, hyperboloid of one sheet, hyperboloid of two sheets, cone, cylinder, single point, empty (no solutions).

(a) $x^2 + y^2 - 3z^2 = 2$

Answer: hyperboloid of one sheet

(b) $3x^2 + 6y^2 - 3z = 0$

Answer: elliptic paraboloid

(c) $2x^2 = 14 - y^2 - 3z^2$

Answer: ellipsoid

(d) $5x^2 + y - 3z^2 = -3$

Answer: hyperbolic paraboloid

(e) $x^2 = y - y^2$

Answer: (elliptic / circular) cylinder